

# Mathematica 11.3 Integration Test Results

Test results for the 93 problems in " $2.2 (c+dx)^m (F^{g(e+fx)})^n (a+b(F^{g(e+fx)})^n)^{p,m}$ "

Problem 46: Unable to integrate problem.

$$\int \frac{(c+dx)^3}{a+b(F^{g(e+fx)})^n} dx$$

Optimal (type 4, 192 leaves, 6 steps):

$$\frac{(c+dx)^4}{4ad} - \frac{(c+dx)^3 \operatorname{Log}\left[1 + \frac{b(F^{g(e+fx)})^n}{a}\right]}{afgn \operatorname{Log}[F]} - \frac{3d(c+dx)^2 \operatorname{PolyLog}\left[2, -\frac{b(F^{g(e+fx)})^n}{a}\right]}{af^2g^2n^2 \operatorname{Log}[F]^2} +$$

$$\frac{6d^2(c+dx) \operatorname{PolyLog}\left[3, -\frac{b(F^{g(e+fx)})^n}{a}\right]}{af^3g^3n^3 \operatorname{Log}[F]^3} - \frac{6d^3 \operatorname{PolyLog}\left[4, -\frac{b(F^{g(e+fx)})^n}{a}\right]}{af^4g^4n^4 \operatorname{Log}[F]^4}$$

Result (type 8, 27 leaves):

$$\int \frac{(c+dx)^3}{a+b(F^{g(e+fx)})^n} dx$$

Problem 47: Unable to integrate problem.

$$\int \frac{(c+dx)^2}{a+b(F^{g(e+fx)})^n} dx$$

Optimal (type 4, 145 leaves, 5 steps):

$$\frac{(c+dx)^3}{3ad} - \frac{(c+dx)^2 \operatorname{Log}\left[1 + \frac{b(F^{g(e+fx)})^n}{a}\right]}{afgn \operatorname{Log}[F]} -$$

$$\frac{2d(c+dx) \operatorname{PolyLog}\left[2, -\frac{b(F^{g(e+fx)})^n}{a}\right]}{af^2g^2n^2 \operatorname{Log}[F]^2} + \frac{2d^2 \operatorname{PolyLog}\left[3, -\frac{b(F^{g(e+fx)})^n}{a}\right]}{af^3g^3n^3 \operatorname{Log}[F]^3}$$

Result (type 8, 27 leaves):

$$\int \frac{(c+dx)^2}{a+b(F^{g(e+fx)})^n} dx$$

**Problem 48: Attempted integration timed out after 120 seconds.**

$$\int \frac{c + dx}{a + b (F^{(e+fx)})^n} dx$$

Optimal (type 4, 98 leaves, 4 steps):

$$\frac{(c + dx)^2}{2 a d} - \frac{(c + dx) \operatorname{Log}\left[1 + \frac{b (F^{(e+fx)})^n}{a}\right]}{a f g n \operatorname{Log}[F]} - \frac{d \operatorname{PolyLog}\left[2, -\frac{b (F^{(e+fx)})^n}{a}\right]}{a f^2 g^2 n^2 \operatorname{Log}[F]^2}$$

Result (type 1, 1 leaves):

???

**Problem 49: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{a + b (F^{(e+fx)})^n} dx$$

Optimal (type 3, 40 leaves, 5 steps):

$$\frac{x}{a} - \frac{\operatorname{Log}[a + b (F^{(e+fx)})^n]}{a f g n \operatorname{Log}[F]}$$

Result (type 3, 100 leaves):

$$\frac{x}{a} - \frac{1}{a f g n \operatorname{Log}[F]} \operatorname{Log}\left[a + b e^{n(-fx \operatorname{Log}[F] + \operatorname{Log}[F^{e+fx}])}\right] (F^{e+fx})^{n - \frac{n(-fx \operatorname{Log}[F] + \operatorname{Log}[F^{e+fx}])}{\operatorname{Log}[F^{e+fx}]}}$$

**Problem 52: Unable to integrate problem.**

$$\int \frac{(c + dx)^3}{(a + b (F^{(e+fx)})^n)^2} dx$$

Optimal (type 4, 388 leaves, 13 steps):

$$\begin{aligned} & \frac{(c + dx)^4}{4 a^2 d} - \frac{(c + dx)^3}{a^2 f g n \operatorname{Log}[F]} + \frac{(c + dx)^3}{a f (a + b (F^{(e+fx)})^n) g n \operatorname{Log}[F]} + \frac{3 d (c + dx)^2 \operatorname{Log}\left[1 + \frac{b (F^{(e+fx)})^n}{a}\right]}{a^2 f^2 g^2 n^2 \operatorname{Log}[F]^2} - \\ & \frac{(c + dx)^3 \operatorname{Log}\left[1 + \frac{b (F^{(e+fx)})^n}{a}\right]}{a^2 f g n \operatorname{Log}[F]} + \frac{6 d^2 (c + dx) \operatorname{PolyLog}\left[2, -\frac{b (F^{(e+fx)})^n}{a}\right]}{a^2 f^3 g^3 n^3 \operatorname{Log}[F]^3} - \\ & \frac{3 d (c + dx)^2 \operatorname{PolyLog}\left[2, -\frac{b (F^{(e+fx)})^n}{a}\right]}{a^2 f^2 g^2 n^2 \operatorname{Log}[F]^2} - \frac{6 d^3 \operatorname{PolyLog}\left[3, -\frac{b (F^{(e+fx)})^n}{a}\right]}{a^2 f^4 g^4 n^4 \operatorname{Log}[F]^4} + \\ & \frac{6 d^2 (c + dx) \operatorname{PolyLog}\left[3, -\frac{b (F^{(e+fx)})^n}{a}\right]}{a^2 f^3 g^3 n^3 \operatorname{Log}[F]^3} - \frac{6 d^3 \operatorname{PolyLog}\left[4, -\frac{b (F^{(e+fx)})^n}{a}\right]}{a^2 f^4 g^4 n^4 \operatorname{Log}[F]^4} \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{(c+dx)^3}{(a+b(F^{g(e+fx)})^n)^2} dx$$

Problem 53: Unable to integrate problem.

$$\int \frac{(c+dx)^2}{(a+b(F^{g(e+fx)})^n)^2} dx$$

Optimal (type 4, 294 leaves, 11 steps):

$$\begin{aligned} & \frac{(c+dx)^3}{3a^2d} - \frac{(c+dx)^2}{a^2fgn \operatorname{Log}[F]} + \frac{(c+dx)^2}{af(a+b(F^{g(e+fx)})^n)gn \operatorname{Log}[F]} + \\ & \frac{2d(c+dx) \operatorname{Log}\left[1 + \frac{b(F^{g(e+fx)})^n}{a}\right]}{a^2f^2g^2n^2 \operatorname{Log}[F]^2} - \frac{(c+dx)^2 \operatorname{Log}\left[1 + \frac{b(F^{g(e+fx)})^n}{a}\right]}{a^2fgn \operatorname{Log}[F]} + \frac{2d^2 \operatorname{PolyLog}\left[2, -\frac{b(F^{g(e+fx)})^n}{a}\right]}{a^2f^3g^3n^3 \operatorname{Log}[F]^3} - \\ & \frac{2d(c+dx) \operatorname{PolyLog}\left[2, -\frac{b(F^{g(e+fx)})^n}{a}\right]}{a^2f^2g^2n^2 \operatorname{Log}[F]^2} + \frac{2d^2 \operatorname{PolyLog}\left[3, -\frac{b(F^{g(e+fx)})^n}{a}\right]}{a^2f^3g^3n^3 \operatorname{Log}[F]^3} \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{(c+dx)^2}{(a+b(F^{g(e+fx)})^n)^2} dx$$

Problem 54: Attempted integration timed out after 120 seconds.

$$\int \frac{c+dx}{(a+b(F^{g(e+fx)})^n)^2} dx$$

Optimal (type 4, 191 leaves, 11 steps):

$$\begin{aligned} & \frac{(c+dx)^2}{2a^2d} - \frac{dx}{a^2fgn \operatorname{Log}[F]} + \frac{c+dx}{af(a+b(F^{g(e+fx)})^n)gn \operatorname{Log}[F]} + \\ & \frac{d \operatorname{Log}[a+b(F^{g(e+fx)})^n]}{a^2f^2g^2n^2 \operatorname{Log}[F]^2} - \frac{(c+dx) \operatorname{Log}\left[1 + \frac{b(F^{g(e+fx)})^n}{a}\right]}{a^2fgn \operatorname{Log}[F]} - \frac{d \operatorname{PolyLog}\left[2, -\frac{b(F^{g(e+fx)})^n}{a}\right]}{a^2f^2g^2n^2 \operatorname{Log}[F]^2} \end{aligned}$$

Result (type 1, 1 leaves):

???

Problem 58: Unable to integrate problem.

$$\int \frac{(c+dx)^3}{(a+b(F^{g(e+fx)})^n)^3} dx$$

Optimal (type 4, 594 leaves, 26 steps):

$$\begin{aligned} & \frac{(c+dx)^4}{4a^3d} + \frac{3d(c+dx)^2}{2a^3f^2g^2n^2\text{Log}[F]^2} - \frac{3d(c+dx)^2}{2a^2f^2(a+b(F^{(g(e+fx)))^n})g^2n^2\text{Log}[F]^2} - \\ & \frac{3(c+dx)^3}{2a^3fgn\text{Log}[F]} + \frac{(c+dx)^3}{2af(a+b(F^{(g(e+fx)))^n})^2gn\text{Log}[F]} + \\ & \frac{(c+dx)^3}{a^2f(a+b(F^{(g(e+fx)))^n})gn\text{Log}[F]} - \frac{3d^2(c+dx)\text{Log}\left[1+\frac{b(F^{(g(e+fx)))^n}{a}\right]}{a^3f^3g^3n^3\text{Log}[F]^3} + \\ & \frac{9d(c+dx)^2\text{Log}\left[1+\frac{b(F^{(g(e+fx)))^n}{a}\right]}{2a^3f^2g^2n^2\text{Log}[F]^2} - \frac{(c+dx)^3\text{Log}\left[1+\frac{b(F^{(g(e+fx)))^n}{a}\right]}{a^3fgn\text{Log}[F]} - \\ & \frac{3d^3\text{PolyLog}\left[2,-\frac{b(F^{(g(e+fx)))^n}{a}\right]}{a^3f^4g^4n^4\text{Log}[F]^4} + \frac{9d^2(c+dx)\text{PolyLog}\left[2,-\frac{b(F^{(g(e+fx)))^n}{a}\right]}{a^3f^3g^3n^3\text{Log}[F]^3} - \\ & \frac{3d(c+dx)^2\text{PolyLog}\left[2,-\frac{b(F^{(g(e+fx)))^n}{a}\right]}{a^3f^2g^2n^2\text{Log}[F]^2} - \frac{9d^3\text{PolyLog}\left[3,-\frac{b(F^{(g(e+fx)))^n}{a}\right]}{a^3f^4g^4n^4\text{Log}[F]^4} + \\ & \frac{6d^2(c+dx)\text{PolyLog}\left[3,-\frac{b(F^{(g(e+fx)))^n}{a}\right]}{a^3f^3g^3n^3\text{Log}[F]^3} - \frac{6d^3\text{PolyLog}\left[4,-\frac{b(F^{(g(e+fx)))^n}{a}\right]}{a^3f^4g^4n^4\text{Log}[F]^4} \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{(c+dx)^3}{(a+b(F^{(g(e+fx)))^n})^3} dx$$

Problem 59: Unable to integrate problem.

$$\int \frac{(c+dx)^2}{(a+b(F^{(g(e+fx)))^n})^3} dx$$

Optimal (type 4, 439 leaves, 24 steps):

$$\begin{aligned} & \frac{(c+dx)^3}{3a^3d} + \frac{d^2x}{a^3f^2g^2n^2\text{Log}[F]^2} - \frac{d(c+dx)}{a^2f^2(a+b(F^{(g(e+fx)))^n})g^2n^2\text{Log}[F]^2} - \frac{3(c+dx)^2}{2a^3fgn\text{Log}[F]} + \\ & \frac{(c+dx)^2}{2af(a+b(F^{(g(e+fx)))^n})^2gn\text{Log}[F]} + \frac{(c+dx)^2}{a^2f(a+b(F^{(g(e+fx)))^n})gn\text{Log}[F]} - \\ & \frac{d^2\text{Log}[a+b(F^{(g(e+fx)))^n}]}{a^3f^3g^3n^3\text{Log}[F]^3} + \frac{3d(c+dx)\text{Log}\left[1+\frac{b(F^{(g(e+fx)))^n}{a}\right]}{a^3f^2g^2n^2\text{Log}[F]^2} - \frac{(c+dx)^2\text{Log}\left[1+\frac{b(F^{(g(e+fx)))^n}{a}\right]}{a^3fgn\text{Log}[F]} + \\ & \frac{3d^2\text{PolyLog}\left[2,-\frac{b(F^{(g(e+fx)))^n}{a}\right]}{a^3f^3g^3n^3\text{Log}[F]^3} - \frac{2d(c+dx)\text{PolyLog}\left[2,-\frac{b(F^{(g(e+fx)))^n}{a}\right]}{a^3f^2g^2n^2\text{Log}[F]^2} + \frac{2d^2\text{PolyLog}\left[3,-\frac{b(F^{(g(e+fx)))^n}{a}\right]}{a^3f^3g^3n^3\text{Log}[F]^3} \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{(c+dx)^2}{(a+b(F^{g(e+fx)}))^3} dx$$

Problem 60: Attempted integration timed out after 120 seconds.

$$\int \frac{c+dx}{(a+b(F^{g(e+fx)}))^3} dx$$

Optimal (type 4, 276 leaves, 17 steps):

$$\begin{aligned} & \frac{(c+dx)^2}{2a^3d} - \frac{d}{2a^2f^2(a+b(F^{g(e+fx)}))^n g^2 n^2 \text{Log}[F]^2} - \frac{3dx}{2a^3fgn \text{Log}[F]} + \\ & \frac{c+dx}{2af(a+b(F^{g(e+fx)}))^n g n \text{Log}[F]} + \frac{c+dx}{a^2f(a+b(F^{g(e+fx)}))^n g n \text{Log}[F]} + \\ & \frac{3d \text{Log}[a+b(F^{g(e+fx)}))^n]}{2a^3f^2g^2n^2 \text{Log}[F]^2} - \frac{(c+dx) \text{Log}[1+\frac{b(F^{g(e+fx)})^n}{a}]}{a^3fgn \text{Log}[F]} - \frac{d \text{PolyLog}[2, -\frac{b(F^{g(e+fx)})^n}{a}]}{a^3f^2g^2n^2 \text{Log}[F]^2} \end{aligned}$$

Result (type 1, 1 leaves):

???

Problem 70: Unable to integrate problem.

$$\int (a+b(F^{g(e+fx)}))^3 (c+dx)^m dx$$

Optimal (type 4, 340 leaves, 8 steps):

$$\begin{aligned} & \frac{a^3(c+dx)^{1+m}}{d(1+m)} + \frac{1}{fgn \text{Log}[F]} 3^{-1-m} b^3 F^{3(e-\frac{cf}{d})} g^{n-3} g^n (e+fx) (F^{eg+fgx})^{3n} (c+dx)^m \\ & \text{Gamma}[1+m, -\frac{3fgn(c+dx) \text{Log}[F]}{d}] \left( -\frac{fgn(c+dx) \text{Log}[F]}{d} \right)^{-m} + \frac{1}{fgn \text{Log}[F]} \\ & 3 \times 2^{-1-m} a b^2 F^{2(e-\frac{cf}{d})} g^{n-2} g^n (e+fx) (F^{eg+fgx})^{2n} (c+dx)^m \text{Gamma}[1+m, -\frac{2fgn(c+dx) \text{Log}[F]}{d}] \\ & \left( -\frac{fgn(c+dx) \text{Log}[F]}{d} \right)^{-m} + \frac{1}{fgn \text{Log}[F]} 3 a^2 b F^{(e-\frac{cf}{d})} g^{n-g} g^n (e+fx) (F^{eg+fgx})^n \\ & (c+dx)^m \text{Gamma}[1+m, -\frac{fgn(c+dx) \text{Log}[F]}{d}] \left( -\frac{fgn(c+dx) \text{Log}[F]}{d} \right)^{-m} \end{aligned}$$

Result (type 8, 27 leaves):

$$\int (a+b(F^{g(e+fx)}))^3 (c+dx)^m dx$$

**Problem 71: Unable to integrate problem.**

$$\int (a + b (F^{g(e+fx)})^n)^2 (c + dx)^m dx$$

Optimal (type 4, 228 leaves, 6 steps):

$$\frac{a^2 (c + dx)^{1+m}}{d (1 + m)} + \frac{1}{f g n \text{Log}[F]}$$

$$2^{-1-m} b^2 F^{2 \left( e - \frac{cf}{d} \right) g n - 2 g n (e+fx)} (F^{e g + f g x})^{2n} (c + dx)^m \text{Gamma} \left[ 1 + m, - \frac{2 f g n (c + dx) \text{Log}[F]}{d} \right]$$

$$\left( - \frac{f g n (c + dx) \text{Log}[F]}{d} \right)^{-m} + \frac{1}{f g n \text{Log}[F]} 2 a b F^{\left( e - \frac{cf}{d} \right) g n - g n (e+fx)} (F^{e g + f g x})^n$$

$$(c + dx)^m \text{Gamma} \left[ 1 + m, - \frac{f g n (c + dx) \text{Log}[F]}{d} \right] \left( - \frac{f g n (c + dx) \text{Log}[F]}{d} \right)^{-m}$$

Result (type 8, 27 leaves):

$$\int (a + b (F^{g(e+fx)})^n)^2 (c + dx)^m dx$$

**Problem 72: Unable to integrate problem.**

$$\int (a + b (F^{g(e+fx)})^n) (c + dx)^m dx$$

Optimal (type 4, 116 leaves, 4 steps):

$$\frac{a (c + dx)^{1+m}}{d (1 + m)} + \frac{1}{f g n \text{Log}[F]} b F^{\left( e - \frac{cf}{d} \right) g n - g n (e+fx)} (F^{e g + f g x})^n$$

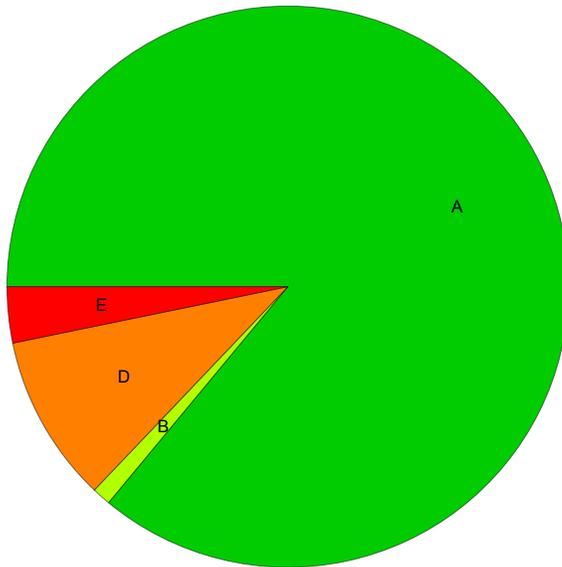
$$(c + dx)^m \text{Gamma} \left[ 1 + m, - \frac{f g n (c + dx) \text{Log}[F]}{d} \right] \left( - \frac{f g n (c + dx) \text{Log}[F]}{d} \right)^{-m}$$

Result (type 8, 25 leaves):

$$\int (a + b (F^{g(e+fx)})^n) (c + dx)^m dx$$

## Summary of Integration Test Results

93 integration problems



A - 80 optimal antiderivatives

B - 1 more than twice size of optimal antiderivatives

C - 0 unnecessarily complex antiderivatives

D - 9 unable to integrate problems

E - 3 integration timeouts